Application Notes/Briefs

TRIG FUNCTION GENERATORS

Accuracy is the major design variable of trigonometric lookup tables built with MOS read-only memories. Only a few ROMs are needed for most practical applications, but accuracy can be made to increase very rapidly with memory capacity if interpolation techniques are used.

For instance, without interpolation a single 1024-bit ROM can store 128 angular increments and generate an 8-bit output that will be better than 99.9% of the handbook value (Table 1).

ADDRESS	DEGREES	BINARY OUTPUT	DECIMAL SINE
0	0	.00000000	0.000
1	0.7	.00000011	0.012
2	1.4	.00000110	0.023
3	2.1	.00001001	0.035
•			
•			
•			
127	89.3	.11111111	0.996

TABLE 1. MM422BM/MM522BM Sine Function Generator

If one simply cascaded ROMs to improve input resolution and output accuracy for a high-accuracy trig solution (X=sin θ) as in Figure 1, large numbers of ROMs might be needed. This 24-ROM system stores 2048 12-bit values of sin x (or other trig functions), giving angular resolution of 1 part in 2^{11} (0.05%) and output accuracy of 1 part in 2^{12} (0.024%). The system in Figure 2 has the same resolution and is accurate to the limit of its 12 output bits (0.024%), which makes it just as good. But it only requires four 1024-bit ROMs and three 4-bit TTL full adders, so it only costs about one-fifth as much as the more obvious solution of Figure 1.

Instead of producing x = sin θ, the Figure 2 system divides the angle into two parts and implements the equation

$$x = \sin\theta = \sin(M + L)$$
$$= \sin M \cos L + \cos M \sin L$$

It can be programmed for any angular range. Assume the range is 0 to 90 degrees and let M be the 8 most significant bits of θ and L be the 3 least significant bits of θ (θ being the 11-bit input angular increments, equal to 90°/2048, or 0.044 deg.) as in Table 2.

With an 8-bit address, the three 256x4 ROMs will give the 12-bit value of sin M at increments of M = 90°/2^8, or 0.352 deg. The cos L can only vary between 1 and 0.99998. So we assume cos L=1 and store values of sin M at 0.352 deg. resolution

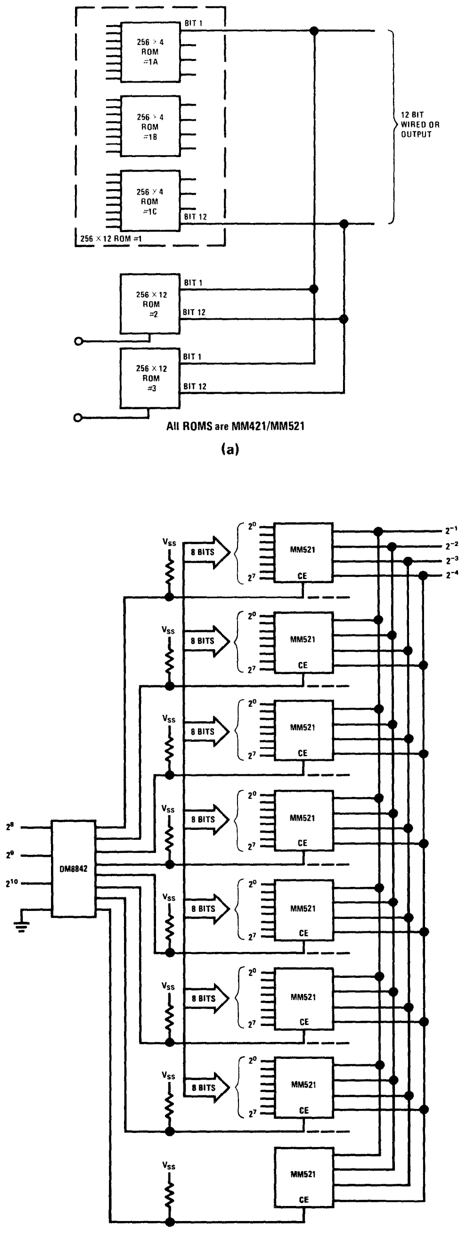

FIGURE 1. Conventional 2048-Increment Sine Table Uses 24 ROMs

in the top three ROMs, reducing the equation to

$$\sin \theta = \sin M + \cos M \sin L$$

Values of the second term are stored in the fourth ROM. The maximum value of the second term in the above equation can only be $\cos M \sin L = 0.00539$ where $\cos M_{max} = 1$, $\sin L_{max} = 0.00539$. This is the maximum value to be added to sin M above. Only the five least significant bits of a 12-bit output are needed to form the maximum output, so an MM522 is used in its 128x8 configuration.

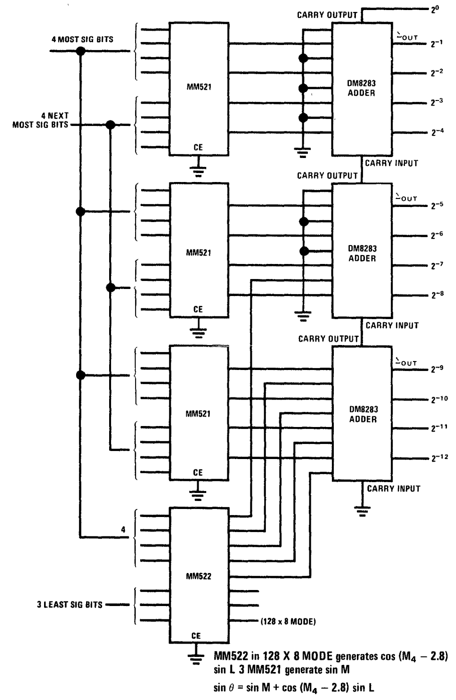

FIGURE 2. Four-ROM Lookup Table Generates 2048 Values of Sin x by Interpolation Technique.

Let the 4 most significant bits of M be called M_4 and the angle at these increments be $X_m = 90°/2^4 = 5.63$ deg. Sin L (the 3 least significant bits of θ) has the same maximum as before and $\cos M_4$ has a maximum of cos 5.63 deg. = 0.99517, and continuing as follows:

$$\cos(11.26) = 0.98076$$

$$\cos(16.89) = 0.95686$$

$$\cos(84.37) = 0.09810$$

through the 16 increments of M_4. Now

$$\sin \theta = \sin M + \cos M_4 \sin L$$

and the appropriate cos M sin L values are stored in the fourth ROM. In effect, we have divided the 0° to 90° sine curve into 16 slope sectors with M_4, each sector into 16 subsections with M, and each subsection into 8 interpolation segments with L.

Since we are using an approximation, accuracy is not quite as good as the Figure 1 system. The additional error term is cos L, assumed 1 but actually is a variable between 1 and 0.99998. At every eighth increment, L is zero, making cos M

ADDRESS	M		L	
	M_4			
0			0	
1			1	L = 0.044°
2			1 0	
3			1 1	
4			1 0 0	
5			1 0 1	
6			1 1 0	
7			1 1 1	
8		1	0 0 0	M = 0.352°
9		1	0 0 1	
16		1 0	0 0 0	
32		1 0 0	0 0 0	
64		1 0 0 0	0 0 0	
128	1	0 0 0 0	0 0 0	M_4 = 5.63°
256	1 0	0 0 0 0	0 0 0	
512	1 0 0	0 0 0 0	0 0 0	
1024	1 0 0 0	0 0 0 0	0 0 0	
2048-1	1 1 1 1	1 1 1 1	1 1 1	

TABLE 2. Programming of 2048-Increment Sine Table

sin L=0, and sin x=sin M to 12-bit accuracy. Then the error rises to a limit of near 0.002% at every eighth increment where L is 0.352–0.044. This error can be halved by adjusting the fourth ROM's output so that

$$\sin \theta = \sin M + \cos(M - 2.81°) \sin L$$

If five ROMs are used—four MM521's and all eight outputs of the MM522—15-bit accuracy can be achieved, and thus improving the accuracy by a factor of eight. The resolution could also be smaller, of course, if the angular range were smaller as in an application involving a sensor with a limited field of view. Variations of the system could be used to space the increments irregularly to compensate for sensor nonlinearities, to improve accuracy in specific angular ranges.

This example has a binary fraction output, like the sine function generator in Table 1. For instance, the 8-bit output at the 64th increment representing $\sin x = \sin 45°$ is 10110101. This equals $1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 0 \times 2^{-5} + 1 \times 2^{-6} + 0 \times 2^{-7} + 1 \times 2^{-8}$, which reduces to 181/256 or 0.7070. Handbooks give the four-place sine of 45° as 0.7071, so at this increment the output is accurate to approximately 0.01%. This table, the MM422BM/MM522BM, is used in fast Fourier transform, radar, and other signal-processing applications.

Other standard tables that are available off the shelf include an arctan generator, several code generators (EBCDIC to ASCII, BCD to Selectric, and Selectric to BCD) and ASCII-addressed character generators for electronic, electrical and electromechanical display and printout systems. All interface with TTL logic and operate off 12-volt power supplies. Write for data sheets, or use one of our programming tables to jot down any special input-output logic functions you need.